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Analytical solution of thermoelastic interaction in a half-space by pulsed laser heating

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ABSTRACT

In this article, we consider the problem of a two-dimensional thermoelastic half-space in the context of generalized thermoelastic theory with one relaxation time. The surface of the half-space is taken to be traction free and thermally insulated. The solution of the considered physical quantity can be broken down in terms of normal modes. The nonhomogeneous basic equations have been written in the form of a vector-matrix differential equation, which is then solved by an eigenvalue approach. The exact analytical solution is adopted for the temperature, the components of displacement and stresses. The results obtained are presented graphically for the effect of laser pulse to display the phenomena physical meaning. The graphical results indicate that the thermal relaxation time has a great effect on the temperature, the components of displacement and the components of stress.

1. Introduction

In the case of ultra-short laser heating, elastic waves propagate long distance in the structures, they provide an effective way to characterize the properties of engineering structures such as thermal properties, elastic. Very rapid thermal processes, under the action of ultra-short laser pulse, are interesting the thermoelasticity viewpoint because they require an analysis of temperature fields and deformation coupled. Irradiation of the surface of a solid by the pulsed laser light generates the movement of the waves in the solid material. This waves have many applications which use in the dimensional properties measurement. For example, the thickness of the plate can be accurately measured by the technique of "time of flight" of a pulsed wave. The waves are not audible to the human ear, because the dominant frequencies of the motion of the waves generated are generally above 2×10^4 Hz, and are therefore called “ultrasonic waves”.

Biot [1] modified the theory of classical coupled thermoelasticity (CTE) to eliminate the paradox inherent in the classical uncoupled thermoelasticity theory that ‘elastic changes has no effect on temperature’. The heat equations for both theories predict infinite speeds of propagation for heat waves. So, various generalized theories of thermoelasticity were developed Lord and Shulman [2] established the first model generalized thermoelasticity theory (LS). Green and Lindsay [3] proposed the temperature rate dependent thermoelasticity (GL) theory. The theory was extended for anisotropic body by Dhaliwal and Sherief [4]. During the second half of twentieth century, a large amount of work has been devoted to solving thermoelastic problems. The counterparts of our problem in the contexts of the thermoelasticity theories have been considered by using analytical and numerical methods [5–20].

Deresiewicz [21] studied under plain strain state, the propagation of waves in thermoelastic plates. Mallik and Kanoria [22] studied the thermoelastic interaction in a transversely isotropic thick plate due to varying heat source. The wave propagation in an isotropic plate under generalized thermoelastic theory has considered by Puri [23]. Verma and Hasebe [24] studied the wave propagation in an infinite thermoelastic plates under Green and Naghdi of type II. A model of the equations of generalized magneto-thermoelasticity in a perfectly conducting medium is given by Ezzat and Youssef [25]. Agarwal [26–28] investigated the wave propagation in generalized thermoelaticity and electromagneto-thermoelastic medium.

In this article, an analytical solution for generalized thermoelastic interaction with one relaxation time on half-space subjected thermal loading due to laser pulse is developed. The nonhomogeneous basic equations of the mathematical model is presented when the surface of the half-space is quiescent first. Also, the surface of the half-space is

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taken to be traction free and thermally insulated. The eigenvalue approach is used to obtain the solutions of non-dimensional nonhomogeneous equations. The theory and numerical study in the present problem may help experimental scientists working in the area of computational wave propagation.

2. Basic equation

Following Lord and Shulman [2], the basic equations for an elastic, isotropic and homogeneous material in the context of generalized thermoelastic theory in the absence of body forces can be considered as.

The equations of motion without body force takes the form

$$\sigma_{ij,j} = \rho \frac{\partial^2 u_i}{\partial t^2}. \tag{1}$$

The heat conduction equations:

$$(K_{ij} T_{,j})_{,i} = \left(1 + \tau_o \frac{\partial}{\partial t}\right)\left(\rho c_e \frac{\partial T}{\partial t} + \gamma T_0 \frac{\partial e_{ii}}{\partial t} - Q\right). \tag{2}$$

The stress-strain relations are given by [2]

$$\sigma_{ij} = 2\mu e_{ij} + [\lambda e_{kk} - \gamma T]\delta_{ij}, \tag{3}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{4}$$

where ρ is the density of the medium, λ, μ are the Lame's constants, c_e is the specific heat at constant strain, $\gamma = (3\lambda + 2\mu)\alpha_t$, α_t is the coefficient of linear thermal expansion. The variable $T = T^* - T_0$ is the temperature increment of the material,T_0 is the reference temperature, t is the time, τ_o is the thermal relaxation time, δ_{ij} is the Kronecker symbol, K is the thermal conductivity. The tensor σ_{ij} are the components of stress tensor, e_{ij} are the components of strain tensor, u_i are the displacement vector components and Q is the internal heat source. We consider an isotropic, homogeneous, generalized thermoelastic half space initially at uniform temperature T_0. The system of Cartesian coordinate (x, y, z) can be used with origin on the surface x = 0, which is stress free and with y-axis directed vertically into the medium. The region x > 0 is occupied by the elastic solid. We assume that all quantities are functions of the coordinates x, y and time $t$and independent of coordinate z. So the components of displacement vector and temperature can be taken in the following form

$$u = u_x = u(x, y, t), \quad v = u_y = v(x, y, t), \quad w = u_z = 0, \quad T = T(x, y, t). \tag{5}$$

Thus, the governing equations for two dimensional problem can be take the form

$$(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2} + (\lambda + \mu)\frac{\partial^2 v}{\partial x \partial y} - \gamma \frac{\partial T}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2}, \tag{6}$$

$$(\lambda + 2\mu)\frac{\partial^2 v}{\partial y^2} + \mu \frac{\partial^2 v}{\partial x^2} + (\lambda + \mu)\frac{\partial^2 u}{\partial x \partial y} - \gamma \frac{\partial T}{\partial y} = \rho \frac{\partial^2 v}{\partial t^2}, \tag{7}$$

$$K\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) = \left(1 + \tau_o \frac{\partial}{\partial t}\right)\left(\rho c_e \frac{\partial T}{\partial t} + \gamma T_0\left(\frac{\partial^2 u}{\partial t \partial x} + \frac{\partial^2 v}{\partial t \partial y}\right) - Q(x, y, t)\right), , \tag{8}$$

$$\sigma_{xx} = (\lambda + 2\mu)\frac{\partial u}{\partial x} + \lambda \frac{\partial v}{\partial y} - \gamma T, \tag{9}$$

$$\sigma_{yy} = (\lambda + 2\mu)\frac{\partial v}{\partial y} + \lambda \frac{\partial u}{\partial x} - \gamma T, \tag{10}$$

$$\sigma_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right). \tag{11}$$

We define the following dimensionless quantities

$$(x', y', u', v') = \frac{\eta}{c_1}(x, y, u, v), \quad (t', \tau_o') = \eta(t, \tau_o), \quad T' = \frac{\gamma T}{\rho c_1^2},$$
$$\eta = \frac{\rho c_e c_1^2}{k},$$
$$(\sigma_{xx}', \sigma_{xy}', \sigma_{yy}') = \frac{1}{\rho c_1^2}(\sigma_{xx}, \sigma_{xy}, \sigma_{yy}), \quad Q' = \frac{\gamma_1}{\rho K \eta^2}Q, \ c_1^2 = \frac{\lambda + 2\mu}{\rho}. \tag{12}$$

In terms of the non-dimensional quantities defined in Eq. (12), the above governing equations reduce to (dropping the dashed for convenience)

$$\frac{\partial^2 u}{\partial x^2} + \beta_1 \frac{\partial^2 u}{\partial y^2} + \beta_2 \frac{\partial^2 v}{\partial x \partial y} - \frac{\partial T}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \tag{13}$$

$$\frac{\partial^2 v}{\partial y^2} + \beta_1 \frac{\partial^2 v}{\partial x^2} + \beta_2 \frac{\partial^2 u}{\partial x \partial y} - \frac{\partial T}{\partial y} = \frac{\partial^2 v}{\partial t^2}, \tag{14}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \left(1 + \tau_o \frac{\partial}{\partial t}\right)\left(T + \varepsilon\left(\frac{\partial^2 u}{\partial t \partial x} + \frac{\partial^2 v}{\partial t \partial y}\right) - Q(x, y, t)\right), \tag{15}$$

$$\sigma_{xx} = \frac{\partial u}{\partial x} + \beta_3 \frac{\partial v}{\partial y} - T, \tag{16}$$

$$\sigma_{yy} = \frac{\partial v}{\partial y} + \beta_3 \frac{\partial u}{\partial x} - T, \tag{17}$$

$$\sigma_{xy} = \beta_1\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \tag{18}$$

where $\beta_1 = \frac{\mu}{\lambda + 2\mu}$, $\beta_2 = \frac{\lambda + \mu}{\lambda + 2\mu}$, $\beta_3 = \frac{\lambda}{\lambda + 2\mu}$, $\varepsilon = \frac{T_0 \gamma^2}{\rho^2 c_1^2 c_e}$.

The surface of half-space is illuminated by laser pulse given by the heat input [29]

$$Q(x, y, t) = I_0 f(t) g(y) h(x), \tag{19}$$

where the temporal profile $f(t)$ is presented as $f(t) = \frac{t}{t_0^2} e^{-\frac{t}{t_0}}$, the pulse is also assumed to have Gaussian profile in y as $g(y) = \frac{1}{2\pi r^2} e^{\frac{-y^2}{r^2}}$ and as a function of the depth x, the heat deposition due to the laser pulse is assumed to decay exponentially within the solid $h(x) = \gamma^* e^{-\gamma^* x}$, where I_0 is the energy absorbed, t_0 is the pulse rise time, r is the beam radius and as a function of the depth x and γ^* is the absorption depth of heating energy. A schematic representation of the pulse is shown in Fig. 1.

3. Initial and boundary conditions

To solve the problem, both the initial and boundary conditions required to be considered. The initial conditions of the problem are assumed to be homogeneous. These initial conditions are complemented by the examination of the surface of the half-space is considered a free traction and thermally insulated:

$$\frac{\partial T(0, y, t)}{\partial x} = 0, \quad \sigma_{xx}(0, y, t) = 0, \quad \sigma_{xy}(0, y, t) = 0. \tag{20}$$

4. Solution of the problem

The solution of the considered physical quantity can be broken down in terms of normal modes as follows:

$$[u, v, T](x, y, t) = [u^*, v^*, T^*](x) e^{(\omega t + imy)}, \tag{21}$$

where m is the wave number in the $y-$ direction, ω is a complex constant, $i = \sqrt{-1}$, and $u^*(x)$, $v^*(x)$ and $T^*(x)$ are the amplitudes of the field quantities.

Thus, the Eqs. (13)–(15), take the form

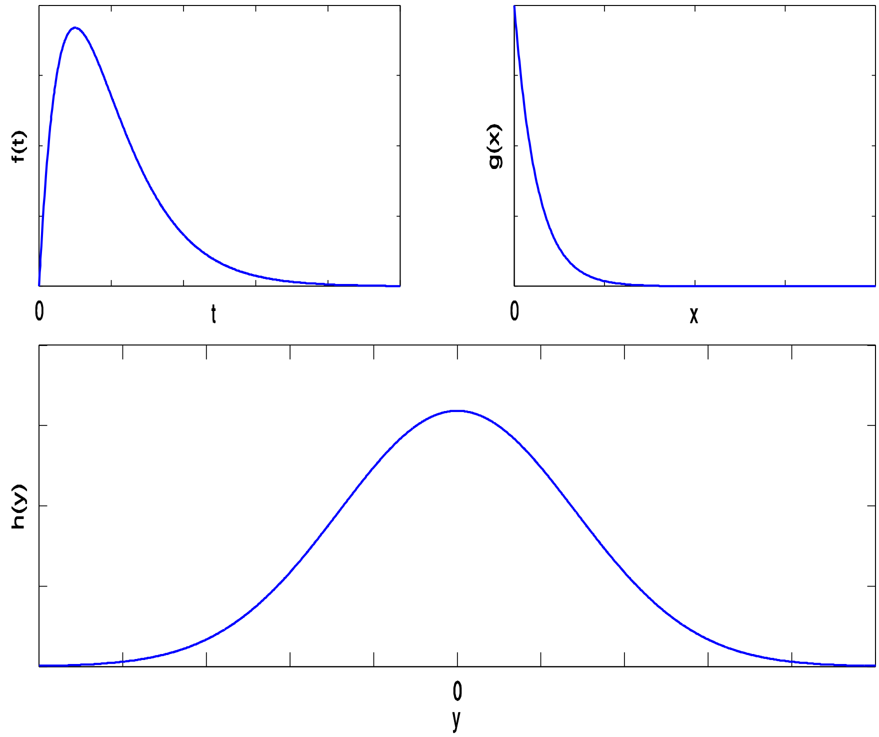

Fig. 1. Temporal and spatial profile of the pulse.

$$\frac{d^2u^*}{dx^2} = a_{41}u^* + a_{45}\frac{dv^*}{dx} + a_{46}\frac{dT^*}{dx}, \tag{22}$$

$$\frac{d^2v^*}{dx^2} = a_{52}v^* + a_{53}T^* + a_{54}\frac{du^*}{dx}, \tag{23}$$

$$\frac{d^2T^*}{dx^2} = a_{62}v^* + a_{63}T^* + a_{64}\frac{du^*}{dx} + J(y,t)e^{-\gamma^* x}, \tag{24}$$

where $a_{41} = \omega^2 + m^2\beta_1$, $a_{45} = -im\beta_2$, $a_{46} = 1$, $a_{52} = \frac{1}{\beta_1}(\omega^2 + m^2)$, $a_{53} = \frac{im}{\beta_1}$, $a_{54} = \frac{-im\beta_2}{\beta_1}$, $a_{62} = im\varepsilon\omega$, $a_{63} = m^2 + \omega(1 + \tau_o\omega)$, $a_{64} = \omega(1 + \tau_o\omega)\varepsilon$

$$J(y,t) = -\frac{I_0\gamma^*}{2\pi r^2 t_0^2}\left(t + \tau_o\left(\frac{t_0 - t}{t_0}\right)\right)e^{\left(-\frac{y^2}{r^2} - \frac{t}{t_0} - \omega t - imy\right)}.$$

The form of vector-matrix differential equation for (22)–(24) can be written in a as follows [30,31]

$$\frac{d\vec{V}}{dx} = A\vec{V} + \vec{J}e^{-\gamma^* x}, \tag{25}$$

where $\vec{V} = [u^* v^* T^* \frac{du^*}{dx}\frac{dv^*}{dx}\frac{dT^*}{dx}]^T$, $\vec{J} = [0\ \ 0\ \ 0\ \ 0\ \ 0\ \ J]^T$ and

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ a_{41} & 0 & 0 & 0 & a_{45} & a_{46} \\ 0 & a_{52} & a_{53} & a_{54} & 0 & 0 \\ 0 & a_{62} & a_{63} & a_{64} & 0 & 0 \end{bmatrix}.$$

The general solutions $\vec{V}$ of the nonhomogeneous system (25) are the sum of the complementary solution $\vec{V_c}$ of the homogeneous equations and a particular solution $\vec{V_p}$ of the nonhomogeneous system. Now, the solutions of homogeneous system obtain by using the eigenvalue approach which proposed by [32]. The matrix A has the characteristic equation in the form

$$R^6 - F_1R^4 + F_2R^2 + F_3 = 0, \tag{26}$$

With

$$F_1 = a_{41} + a_{52} + a_{45}a_{54} + a_{63} + a_{46}a_{64},$$

$$F_2 = a_{41}a_{52} - a_{53}a_{62} - a_{46}a_{54}a_{62} + a_{41}a_{63} + a_{52}a_{63} + a_{45}a_{54}a_{63} + a_{46}a_{52}a_{64} - a_{45}a_{53}a_{64},$$

$$F_3 = a_{41}a_{53}a_{62} - a_{41}a_{52}a_{63}.$$

The characteristic Eq. (26) have six roots which written in the form

$$R = \pm R_1,\ R = \pm R_2,\ R = \pm R_3. \tag{27}$$

The eigenvector $\vec{Y} = [Y_1, Y_2, Y_3, Y_4, Y_5, Y_6]^T$, corresponding to eigenvalue R can be calculated as

$$Y_1 = a_{46}R(a_{52} - R^2) - a_{45}a_{53}R, \tag{28}$$

$$Y_2 = a_{53}(a_{41} - R^2) - a_{46}a_{54}R^2, \tag{29}$$

$$Y_3 = a_{41}(R^2 - a_{52}) + R^2(a_{52} + a_{45}a_{54} - R^2), \tag{30}$$

$$Y_4 = RY_1,\ Y_5 = RY_2,\ Y_6 = RY_3. \tag{31}$$

Now, we can easily calculate the eigenvector $\vec{Y_j}$, corresponding to eigenvalue R_j, $j = 1, 2, \ldots,6$. For further reference, we shall use the following notations:

$$\vec{Y_1} = [\vec{Y}]_{R=-R_1},\quad \vec{Y_2} = [\vec{Y}]_{R=-R_2},\quad \vec{Y_3} = [\vec{Y}]_{R=-R_3},$$
$$\vec{Y_4} = [\vec{Y}]_{R=R_1},\quad \vec{Y_5} = [\vec{Y}]_{R=R_2},\quad \vec{Y_6} = [\vec{Y}]_{R=R_3}. \tag{32}$$

The complementary solution of Eq. (25) can be written from as follows:

$$\vec{V_c} = \sum_{j=1}^{3} A_j\vec{Y_j}e^{-R_j x}, \tag{33}$$

where the terms containing exponentials of growing nature in the space variable x have been discarded due to the regularity condition of the solution at infinity, A_1, A_2 and A_3 are constants to be determined from the boundary condition of the problem. We now turn to finding a particular solution$\vec{V_p}$ of the nonhomogeneous Eq. (25). The inhomogeneous terms in (25) contain the exponential function $e^{-\gamma^* x}$, which coincides with the exponential function in the solution of the homogeneous equation. Therefore, the particular solution $\vec{V_p}$ should be